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Transport phenomenology for a holon–spinon fluid

Derek K K Lee†§ and Patrick A Lee‡

† Theoretical Physics, University of Oxford, 1 Keble Rd, Oxford OX1 3NP, UK

‡ Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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Abstract. We propose that the normal-state transport in the cuprate superconductors can be understood in terms of a two-fluid model of spinons and holons. In our scenario, the resistivity is determined by holon dynamics while magnetotransport involves the recombination of holons and spinons to form physical electrons. Our model implies that the Hall transport time, as defined by Anderson and Ong, is a measure of the electron lifetime, which is shorter than the longitudinal transport time. We predict a strong increase in linewidth with increasing temperature in photoemission. Our model also suggests that the AC Hall effect is controlled by the transport time.

1. Introduction

The normal state of the cuprate superconductor exhibits anomalous transport properties [1]. In this paper, we discuss the implications of the experimental results for theoretical ideas based on spin–charge separation in this system. In our previous work [2], we have argued that transport in an electric field may be described by a boson-only theory for the charge degree of freedom. In this paper, we propose that transport in a magnetic field, such as the Hall effect and magnetoresistance, is controlled by the degree of spin–charge recombination in the system.

We review here the experimental results which provide severe constraints on possible theories. We focus on the case of optimal doping where the superconducting transition temperature T_c is highest. The in-plane resistivity is linear in temperature T. The relaxation rate, measured from a Drude-like peak in the optical conductivity, appears to be universal [3–5]:

$$\hbar/\tau_{tr} \simeq 2k_B T. \tag{1}$$

The spectral weight under the Drude-like peak (or derived from the London penetration depth) is proportional to the hole doping x and can be written as e^2x/ma^2, where a is the lattice constant and m is found to be close to twice the free-electron mass. In a tight-binding model, this mass corresponds to a hopping integral of 1540 K. This is close to the antiferromagnetic exchange J but can also be interpreted as $t/3$. The latter interpretation is consistent with recent studies of the t–J model [6]. The Hall coefficient, on the other hand, is found to be suppressed from the classical value a^2/xec [7]. It rises as $1/T$ with decreasing T, approaching the classical value only near T_c. The magnetoresistance [8] is also strongly suppressed from the expectation that $\Delta\rho/\rho$ scales as τ_{tr}^2.

§ Permanent address: Blackett Laboratory, Imperial College, Prince Consort Rd, London SW7 2BZ, UK.

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There has been remarkable success in analysing the transport data [1, 7, 8] by introducing a new Hall timescale $\tau_{\rm H}$, as suggested by Anderson [9]. In this picture, the Hall angle $\theta_{\rm H} = \sigma_{xy}/\sigma_{xx}$ and $\Delta\rho/\rho$ are given by

$$\tan\theta_{\rm H} = \omega_c\tau_{\rm H} \qquad \text{and} \qquad \Delta\rho/\rho \simeq (\omega_c\tau_{\rm H})^2 \tag{2}$$

with

$$\hbar/\tau_{\rm H} \simeq T^2/W_{\rm H} \tag{3}$$

where $\omega_c = eB/mc$ is the cyclotron frequency and $W_{\rm H}$ is a temperature scale discussed below. In the presence of non-magnetic Zn impurities [10], $1/\tau_{\rm H}$ extrapolates to a finite value at zero temperature but its T-dependence is unaffected. This residual value is proportional to the impurity concentration and is comparable to its longitudinal counterpart. This suggests that $1/\tau_{\rm H}$ is more than a fitting parameter and represents a physical process.

Consider now the temperature scale $W_{\rm H}$. For 90 K YBCO, $\theta_{\rm H}^2 = DB^2/T^4$ where $D = 1630$ K^4 T^{-2} [8]. From (2), we see that the Hall angle measures $\tau_{\rm H}/m$. Using the mass extracted from the optical spectral weight, we obtain $W_{\rm H} = (\hbar/2ea^2)(k_{\rm B}D^{1/2}/J) \simeq 65$ K for 90 K YBCO. The Hall time $\tau_{\rm H}$ is therefore *shorter* than the longitudinal time $\tau_{\rm tr}$ above 130 K, i.e. throughout the whole of the normal state except for the region close to $T_{\rm c}$. Under the assumption of a single mass, the Hall coefficient is

$$R_{\rm H} \simeq \frac{a^2}{xec}\frac{\tau_{\rm H}}{\tau_{\rm tr}} \tag{4}$$

so its reduction from the classical value is direct evidence that $\tau_{\rm H} < \tau_{\rm tr}$.

We note that this analysis is different from the original analysis of references [7, 9], which assumes that $\tau_{\rm H}$ is controlled by the decay of a long-lived quasiparticle ($W_{\rm H} \simeq J$) and so it is longer than the transport time $\tau_{\rm tr}$. As recognized by these authors, this leads one to deduce a carrier mass 20 times larger than the one used above.

Anderson has emphasized that the appearance of $\tau_{\rm H}$ may be a signature of spin–charge separation. There have been attempts to derive (3) [11–14]. Most are based on the Boltzmann transport of a single carrier with unusual scattering mechanisms. For example, Coleman *et al* [13] introduce a mechanism which does not conserve particle number. Kotliar *et al* [14] use a skew-scattering rate which diverges at low temperatures, and $\tau_{\rm H}$ appears not as a physical rate but as a ratio of two rates, so its behaviour with impurity is difficult to rationalize. In this paper, we abandon the notion of a single carrier, and explore a phenomenology based on spin–charge separation.

2. Spin–charge separation

We review first the picture of spin–charge separation in the t–J model which, we believe, describes the low-energy physics of the cuprates. In the slave-boson treatment [15, 16], the introduction of a physical hole (of spin σ at site i) away from half-filling is represented as the creation of a charged hard-core boson (holon) and the destruction of a neutral spin-half fermion (spinon):

$$c_{i\sigma} = b_i^\dagger f_{i\sigma} \tag{5}$$

with a single-occupancy constraint: $b_i^\dagger b_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1$. For a doping of x holes per site, the holon and spinon densities are $n_b = x$ and $n_f = 1 - x$ respectively. In the uniform resonating-valence-bond *ansatz*, short-range antiferromagnetic correlations are incorporated into the model by assuming that $\sum_\sigma \langle f_{i\sigma}^\dagger f_{j\sigma}\rangle = \xi {\rm e}^{{\rm i}a_{ij}}$. At the mean-field level, there is no net gauge flux ($a_{ij} = 0$), so the holons have a bandwidth controlled by the hopping integral t of the original electrons while the spinon bandwidth is controlled by the antiferromagnetic exchange J. In this paper, we will focus on the cuprates near optimal doping where this slave-boson scheme is believed to apply. For instance, it gives rise to a large Fermi surface, as observed in photoemission experiments.

The fluctuations in the gauge field a_{ij} are strong. For temperatures above the experimental T_c, the transverse part of the fluctuations corresponds to a magnetic field with a root mean square value of the order of a flux quantum per plaquette [2]. These fluctuations arise because the single-occupancy constraint requires that the spinon and holon number currents cancel each other:

$$\boldsymbol{J}_f + \boldsymbol{J}_b = 0. \tag{6}$$

At sufficiently low temperatures, the bosons become phase coherent, leading to well-defined physical electron quasiparticles, i.e. spinon–holon confinement or the breakdown of spin–charge separation. We believe that this confinement occurs at T_c, consistently with the fact that the electronic quasiparticles are long-lived in the superconducting state [17].

The fluctuations of the gauge field have a drastic effect on the transport properties of the system. Longitudinal transport should be dominated by the dynamics of the charged holons. The holons are strongly scattered by the gauge fields which can be regarded as quasistatic disorder at long wavelengths and low temperatures. We have shown in a quantum Monte Carlo study [2] of a holon-only model that this gives rise to a holon scattering rate equal to $2k_B T$ which should also be the scattering rate relevant to longitudinal transport. This is consistent with the relaxation rate (1) deduced from the optical conductivity.

The picture that emerged from our previous study [2] is that, in the normal state, the boson de Broglie wavelength is much larger than the interparticle spacing, so the bosons undergo strong exchange and should be viewed as a quantum liquid rather than single particles. The strong gauge field forces the boson world lines to retrace each other, and prevents the development of a superfluid density. In other words, the bosons attempt to avoid the frustrating effects of a random gauge field by retracing each other's paths, and hence they do not detect any random Aharonov–Bohm phase. Consequently, the holons are also insensitive to weak *external* fields. Therefore the holon fluid itself has negligible Hall effect and magnetoconductivity.

We next argue that the spinon fluid also gives a negligible contribution to the magnetic response. This is because the external gauge field couples only to the holon fluid. Therefore, the spinons will contribute to the magnetic response only if the holons develop an orbital current which, via the constraint (6), drives a spinon orbital current. This means that the spinon contribution to the Hall effect scales with the orbital susceptibility χ_b of the holon fluid. More quantitatively, in the random-phase approximation, the total Hall coefficient of the spinon–holon fluid is given by the Ioffe–Larkin rule [15, 16]:

$$R_H = \frac{\chi_f R_{H,b} + \chi_b R_{H,f}}{\chi_f + \chi_b} \tag{7}$$

where $R_{H,b}$ and χ_b are the holon Hall coefficient and orbital susceptibility and $R_{H,f}$ and χ_f are the corresponding spinon quantities. The retracing-path scenario means that, as with the boson Hall response, the boson orbital susceptibility χ_b is suppressed by the gauge fluctuations [2]. Therefore, we see that the spinon contribution to the total Hall response of the system is also small.

We regard this small magnetic response of a system with complete spin–charge separation as a good starting point because qualitatively the anomaly is a *suppression* of the

Hall effect and the magnetoresistance in the cuprates. We now need to find a mechanism to restore some of the response to a magnetic field, especially as the temperature is reduced towards T_c.

It is possible that the self-retracing approximation breaks down due to gauge-field dynamics or a reduction in gauge amplitude, and hence that the response to a magnetic field is gradually restored at low temperatures. In this paper, we explore another possibility. We suggest that the retracing picture remains valid down to T_{c}, and hence that the magnetic response is beyond the scope of the model which we have been discussing so far, i.e. one with complete spin–charge separation. Instead, we propose that the magnetic response could be understood in terms of the incipient recombination of the holons and spinons to form a physical hole. This is based on the observation that a physical hole does not experience any fictitious gauge fields, and hence that its magnetic response should not be suppressed. Even though the hole is made up of a spinon and a holon, the Aharonov–Bohm phases of the holon and the spinon due to the internal gauge field now cancel each other, and the physical hole is not forced to be self-retracing by the internal field. We believe that this recombination increases with decreasing temperature, and that this process is complete when T_c is reached. This is consistent with the observation that the Hall coefficient of the cuprates approaches the classical value as one approaches T_{c}.

3. Spin–charge recombination

We will now develop a simple phenomenology for normal-state transport based on the ideas outlined in the previous section. We do not claim to have the final answer, because we are forced to make a number of assumptions before we can arrive at (3). We can only give some indication of how one might justify some of the assumptions in terms of the gauge theory of the t–J model.

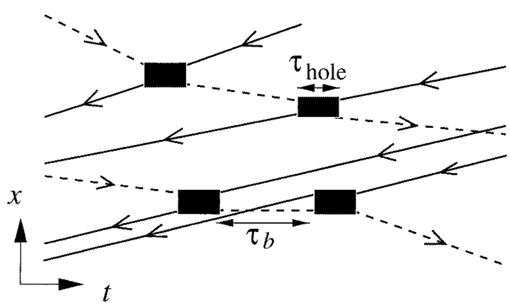

Figure 1. A schematic picture of recombination and decay involving spinons (solid line) and holons (dashed line) and physical holes (box). A hole lives for a time τ_{hole}, shorter than the holon lifetime τ_b and the spinon lifetime τ_b/x. Only the physical hole experiences an external magnetic field.

Our phenomenological picture is as follows. In the normal state the charge carrier exists in two states—either as a holon or as a physical hole:

$$\text{holon (b)} + \text{antispinon } (\bar{\text{f}}) \rightleftharpoons \text{hole (h)}. \tag{8}$$

The densities of the holons n_b, spinons n_f and physical holes n_{hole} obey: $n_b + n_{\mathrm{hole}} = x$ and $n_f + n_b = 1$ where x is the doping per site. The carrier exists as a bosonic holon for a time interval of the order of τ_b before recombining with an antispinon to form a physical hole. This physical hole has a lifetime of $\tau_{\rm hole}$ before decaying back into its spin and charge components. The evolution in space and time of a few holons, spinons and physical holes is shown in figure 1. Detailed balance at equilibrium gives

$$\frac{n_b n_f}{\tau_b} = \frac{n_{\rm hole}}{\tau_{\rm hole}}. \tag{9}$$

In a regime of strong spin–charge separation, the electron lifetime is short ($\tau_{\rm hole} \ll \tau_b$), and hence $n_{\rm hole} \ll n_f, n_b$ and

$$n_{\rm hole} \simeq x\frac{\tau_{\rm hole}}{\tau_b}. \tag{10}$$

As one approaches the confinement regime, $\tau_{\rm hole}$ becomes larger than τ_b and $n_{\rm hole} \simeq n_b$.

We will now discuss the implications of this scenario for transport properties. As mentioned above, the charge carrier responds to external magnetic fields only as a physical hole. In the simplest possible model for this picture, the drift velocity $\boldsymbol{v}$ of the system under an in-plane electric field $\boldsymbol{E}$ and perpendicular magnetic field $\boldsymbol{B}$ obeys the following dynamics:

$$m\dot{\boldsymbol{v}} + \frac{m\boldsymbol{v}}{\tau_{\rm tr}} = e\boldsymbol{E} + \frac{e}{c}\eta(t)\boldsymbol{v}\times\boldsymbol{B}. \tag{11}$$

We assume here a single relaxation time for momentum relaxation which is consistent with experiments at frequencies below $2k_{\rm B}T/\hbar$. The random function $\eta(t)$ is zero except for spikes of value unity and duration $\tau_{\rm hole}$. These spikes occur at time intervals of the order of τ_b. Thus, the response of the charge carriers to an external magnetic field is switched on for a duration of $\tau_{\rm hole}$ and switches off for a duration of $\tau_b \gg \tau_{\rm hole}$, corresponding to the deconfined and confined states in (8) respectively.

At timescales greater than τ_b, the system sees an effective magnetic field $\bar{\eta}\boldsymbol{B}$, where the reduction factor $\bar{\eta} = \tau_{\rm hole}/\tau_b$ is the time-averaged value of η. This can be seen from the classical dynamics of the system. An electric field E in the x-direction accelerates a particle for a duration of $\tau_{\rm tr}$ before the particle velocity is randomized. Thus, the drift velocity of the system is $v_x \sim eE\tau_{\rm tr}/m$, and $\sigma_{xx} = ne^2\tau_{\rm tr}/m$ where m is the holon mass in the spin–charge-separated regime. In this time interval, a particle also receives on average $\tau_{\rm tr}/\tau_b$ impulses of $ev_xB\tau_{\rm hole}/c$ in the y-direction due to the Lorentz force. The transverse drift momentum is $mv_y \sim (ev_xB/c)(\tau_{\rm tr}/\tau_b)\tau_{\rm hole} = eE\omega_c\tau_{\rm tr}^2\tau_{\rm hole}/\tau_b$. From this, we deduce a Hall angle of $\theta_{\rm H} \simeq v_y/v_x = \omega_c\tau_{\rm H}$ where

$$\tau_{\rm H} \simeq \frac{\tau_{\rm hole}}{\tau_b}\tau_{\rm tr}. \tag{12}$$

We therefore see that the Hall effect is reduced from the Fermi-liquid result ($\tau_{\rm H} = \tau_{\rm tr}$) by the fraction

$$\bar{\eta} = \frac{\tau_{\rm hole}}{\tau_b} \simeq \frac{n_{\rm hole}}{n_b}. \tag{13}$$

One can also see in this picture that the x-component of the Lorentz force gives rise to a negative magnetoconductivity proportional to $(\omega_c\tau_{\rm H})^2$.

It should be noted that the simple model (11) does not involve separate decay rates for the longitudinal and transverse drift velocities, so the width $1/\tau_{\rm H}^{\rm ac}$ of the AC Hall angle $\theta_{\rm H}(\omega)$ is given by the transport relaxation time $1/\tau_{\rm tr}$ rather than $1/\tau_{\rm H}$:

$$\theta_{\rm H}(\omega) = \frac{\bar{\eta}(T)eB/mc}{{\rm i}\omega - \tau_{\rm tr}^{-1}}. \tag{14}$$

In our view, the reduction of the magnetic field response is characterized by a reduction of the average charge, rather than by a change in the lifetime. This will also imply a reduction in the Hall sum rule, a point to which we shall return later.

In order not to spoil the description of τ_{tr} as being due to gauge-field scattering [2], it is important to show that the binding of a holon with an antispinon and the subsequent decay of the physical hole do not provide an additional mechanism for dissipating momentum from the holon–spinon system. Note that this requires only the *total* drift current to be conserved during recombination and decay. At first sight this seems unlikely to happen. From the view of the holon which carries a small momentum, it would appear that its momentum is strongly affected in each decay and recombination process. However, from the view of the spinon, which carries a large momentum of order π/a, it is reasonable that it is scattered mainly in the forward direction and that its velocity is preserved. This is indicated in figure 1. We can now appeal to the current constraint (6) to argue that, *on average*, the boson current is also conserved. The constraint is relaxed locally in a spin–charge-separated system, but must remain in force on larger length scales. Another way of saying this is that the depiction of individual scattering events in figure 1 is misleading because the holons are strongly overlapping and exchanging and should not be viewed as individually scattered particles.

So far, our model indicates $1/\tau_H$ to be a combination of three timescales. However, as already pointed out, $1/\tau_H$ changes with disorder as though it is a physical scattering rate. This forces us to conjecture further that

$$\tau_b \sim \tau_{tr} \tag{15}$$

and hence that

$$\tau_H \sim \tau_{hole}. \tag{16}$$

In other words, the Hall transport time is a measure of the lifetime of the physical hole. This provides another important test of our model. The hole lifetime can be deduced independently from angle-resolved photoemission linewidths which we predict from equations (3) and (16) to grow as T^2/W_H. The small size of W_H leads to a severe broadening at room temperature which should be amenable to experimental verification.

The assumption that $\tau_b \sim \tau_{tr}$ is the weakest point of our argument. The only justification that we can offer is that, due to the mismatch of the kinematics of the spinon and the holon, the recombination process is perhaps controlled by the same momentum relaxation process as contributes to τ_{tr}. We also need to argue that, in the presence of impurities, $1/\tau_{hole}$ becomes $1/\tau_{hole} + 1/\tau_0$ where $1/\tau_0$ is a residual value due to impurity scattering. This is not unreasonable in that a hole may well disintegrate rapidly on encountering an impurity. Finally, we can offer no explanation of why τ_{hole} should scale as $1/T^2$. More importantly, we do not understand the origin of W_H which is small and not very sensitive to doping. It is also puzzling that the spin gap in the underdoped cuprates has a much smaller effect on τ_H than on τ_{tr}.

4. Conclusion

We conclude by discussing some experimental consequences of our model. We have already pointed out that our picture predicts a T^2-broadening of photoemission linewidths. Our model, in particular equation (14), also has implications for the AC Hall effect [17–19]. For YBCO film on $LaAlO_3$ [19], it was found at 100 K that the Hall dynamics is characterized by $(\tau_H^{ac})^{-1} \simeq 77$ cm^{-1} while $\tau_{tr}^{-1} \simeq 130$ cm^{-1}. Additional measurements at 28.2 and

54.1 cm^{-1} were reported up to 140 K [19]. By fitting the data to a two-lifetime model, which predicts $\mathrm{Im}\,\theta_{\mathrm{H}} = \omega\omega_c\tau_{\mathrm{H}}^2$ for $\omega\tau_{\mathrm{H}} \ll 1$, τ_{H}^{-1} was found to increase slightly faster than T^2. We note that, from (14), we predict instead

$$\mathrm{Im}\,\theta_{\mathrm{H}} = \omega\omega_c\bar{\eta}\tau_{\mathrm{tr}}^2 \sim \omega\omega_c\tau_{\mathrm{H}}\tau_{\mathrm{tr}} \tag{17}$$

giving an effective rate of $(\tau_{\mathrm{H}}\tau_{\mathrm{tr}})^{-1/2} \sim T^{3/2}$ instead of T^2 in the analysis of [19]. We believe that a wider temperature range is needed to distinguish between the two models. Our model also implies a T-dependent violation of the Hall angle sum rule [19] in that

$$\int \mathrm{Re}\,\theta_{\mathrm{H}}\ \mathrm{d}\omega = \bar{\eta}\,\frac{\pi\omega_c}{2}. \tag{18}$$

The Hall sum is indeed reduced at 95 K compared to that of the superconducting state [19], which we find encouraging. We expect the full sum rule (with $\bar{\eta} = 1$ in the equation above) to be recovered if the integral extends over a much larger energy range (J or t), for which spin–charge separation is no longer relevant.

In summary, we have put forward a hypothesis for understanding transport in the cuprates, based on the idea of spin–charge reconfinement. This model, represented by the two main assumptions (11) and (15), explains naturally the suppression of the Hall response compared to Drude theory. It also links the Hall transport time τ_{H} in DC measurements to the lifetime of a physical hole which can be independently deduced from photoemission linewidths. In this picture, the dynamical time extracted from the AC Hall effect is however the longitudinal scattering time τ_{tr}. We stress that this model (11) is illustrative and ignores the quantum degeneracy of the charge carriers. We hope that, while incomplete, it will stimulate further experimental work and serve as the basis for further discussion.

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References

[1] For a review, see
Ong N P, Yan Y F and Harris J M 1995 *Proc. CCAST Symp. on High-T_c Superconductivity and the C_{60} Family (Beijing, 1994)* (New York: Gordon and Breach)
[2] Lee D K K, Kim D H and Lee P A 1996 *Phys. Rev. Lett.* **76** 4801
Kim D H, Lee D K K and Lee P A 1997 *Phys. Rev.* B **55** 591
[3] Orenstein J, Thomas G A, Millis A J, Cooper S L, Rapkine D H, Timusk T, Schneemeyer L F and Waszczak J V 1990 *Phys. Rev.* B **42** 6342
[4] Gao F, Romero D B, Tanner D B, Talvacchio J and Forrester M G 1993 *Phys. Rev.* B **47** 1036
[5] Romero D B, Porter C D, Tanner D B, Forro L, Mandrus D, Mihaly L, Carr G L and Williams G P 1992 *Phys. Rev. Lett.* **68** 1590
[6] Jaklič J and Prelovšek P 1995 *Phys. Rev.* B **52** 6903
[7] Chien T R, Brawner D A, Wang Z Z and Ong N P 1991 *Phys. Rev.* B **43** 6242
[8] Harris J M, Yan Y F, Matl P, Ong N P, Anderson P W, Kimura T and Kitazawa K 1995 *Phys. Rev. Lett.* **75** 1391
[9] Anderson P W 1991 *Phys. Rev. Lett.* **67** 2092
[10] Chien T R, Wang Z Z and Ong N P 1991 *Phys. Rev. Lett.* **67** 2088
[11] Weng Z Y, Sheng D N and Ting C S 1995 *Phys. Rev.* B **51** 636
[12] Stojković B P and Pines D 1996 *Phys. Rev. Lett.* **76** 811
[13] Coleman P, Schofield A J and Tsvelik A M 1996 *Phys. Rev. Lett.* **76** 1324
[14] Kotliar G, Sengupta A and Varma C M 1996 *Phys. Rev.* B **53** 3573
[15] Ioffe L B and Larkin A I 1989 *Phys. Rev.* B **39** 8988

[16] Nagaosa N and Lee P A 1990 *Phys. Rev. Lett.* **64** 2450
Nagaosa N and Lee P A 1992 *Phys. Rev.* B **46** 5621
[17] Spielman S, Parks B, Orenstein J, Nemeth D T, Ludwig F, Clarke J, Merchant P and Lew D J 1994 *Phys. Rev. Lett.* **73** 1537
Parks B and Orenstein J, private communication
[18] Kaplan S G, Wu S, Lihn H-T S, Drew H D, Li Q, Fenner D B, Phillips J M and Hou S Y 1996 *Phys. Rev. Lett.* **76** 696
[19] Drew H D, Wu S and Lihn H-T S 1996 *J. Phys.: Condens. Matter* **8** 10 036